

# Chirality sensitive effect on surface states in chiral $p$ -wave superconductors

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We study the local density of states at the surface of a chiral  $p$ -wave superconductor in the presence of a weak magnetic field. As a result, the formation of low-energy Andreev bound states is either suppressed or enhanced by an applied magnetic field, depending on its orientation with respect to the chirality of the  $p$ -wave superconductor. Similarly, an Abrikosov vortex, which is situated not too far from the surface, leads to a zero-energy peak of the density of states, if its chirality is the same as that of the superconductor, and to a gap structure for the opposite case. We explain the underlying principle of this effect and propose a chirality sensitive test on unconventional superconductors.

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Much attention has been paid to unconventional superconductors, because they can exhibit a sign or general phase change of their gap function as a function of momentum. This property induces many intriguing phenomena, which can be observed directly by so-called phase sensitive experiments providing powerful tools to test the symmetry of the gap function [1]. One important consequence of the sign change of the gap function is the possible existence of Andreev bound states at the surface of the superconductor [2, 3, 4]. The formation of Andreev bound states increases the local zero-energy quasiparticle density of states (DOS) at the surface, leading to a pronounced zero-bias conductance peak in the tunneling conductance observable both in singlet  $d$ -wave superconductors like the cuprates and in triplet  $p$ -wave superconductors such as  $\text{Sr}_2\text{RuO}_4$  [5, 6, 7, 8, 9, 10, 11, 12, 13]. For the case of  $d$ -wave superconductors it is well-known, that an applied magnetic field or an applied electric current result in a split of this zero-bias conductance peak, since the zero-energy spectral weight of the bound states is effectively Doppler shifted towards higher energies [6, 14, 15, 16]. The same effect also appears for an Abrikosov vortex, which is pinned not too far from the boundary. Here, the zero-energy DOS is suppressed in a shadow-like region 'behind' the vortex [17, 18].

Regarding the chiral  $p$ -wave superconducting phase as it is likely realized in  $\text{Sr}_2\text{RuO}_4$ , a further aspect appears. The chiral  $p$ -wave state characterized by the vector  $\mathbf{d}(\mathbf{k}) = (0, 0, k_x \pm ik_y)$  breaks time reversal symmetry [19, 20, 21, 22, 23]. In this Letter we will show that the influence of an external magnetic field on the surface density of states is selective for the chirality. The quasiparticle density of states at the surface increases or decreases depending on the relative orientation of the applied magnetic field and the chirality. Similarly, we find that the influence of a vortex on the surface states depends on the orientation of vorticity with respect to chirality. These characteristic effects could open an alternative way to chirality sensitive probes, in contrast to

phase or spin sensitive setups, which have been intensively used in the scientific community already.

For our calculations, we use quasiclassical Eilenberger theory of superconductivity [24, 25] in the so-called Riccati-parametrization [26], which allows to achieve numerically stable solutions for the quasiclassical propagators (and thus also for the DOS) in spatially nonhomogeneous systems. Concretely, in our case we consider a superconducting half space  $x > 0$  exhibiting a gap function of chiral  $p$ -wave symmetry. The surface at  $x = 0$  is included in our calculations in a straightforward way by specific boundary conditions [27, 28, 29]. We assume a cylindrical Fermi surface for the superconductor with the symmetry axis pointing along the  $z$ -direction, so that the Fermi velocity can be parametrized by the polar angle  $\theta$  via  $\mathbf{v}_F = v_F(\hat{\mathbf{x}} \cos \theta + \hat{\mathbf{y}} \sin \theta)$ . For a given point  $\mathbf{r}_0$  in space and angle  $\theta$  parametrizing the Fermi surface, a quasiclassical trajectory is then defined according to  $\mathbf{r}(x') = \mathbf{r}_0 + x' \hat{\mathbf{v}}_F$ . Along such a trajectory the Eilenberger equations can be transformed into  $2 \times 2$  matrix differential equations in spin space, which are of the Riccati type and can be solved much easier [29]. In our case, we deal with a one-component gap-function, so that the corresponding Riccati equations take the simpler form [10, 26]

$$\begin{aligned} \hbar v_F \partial_{x'} a(x') + [2\tilde{\epsilon}_n + \Delta^\dagger a(x')] a(x') - \Delta &= 0 \\ \hbar v_F \partial_{x'} b(x') - [2\tilde{\epsilon}_n + \Delta b(x')] b(x') + \Delta^\dagger &= 0 \end{aligned} \quad (1)$$

for the two scalar coherence functions  $a$  and  $b$ . Here,  $i\tilde{\epsilon}_n = i\epsilon_n + \mathbf{v}_F \cdot \frac{\mathbf{e}}{c} \mathbf{A}$  denotes Matsubara frequencies which are shifted due to the presence of a magnetic vector potential  $\mathbf{A}$ . The pairing potential  $\Delta$  can be factorized in the following form

$$\Delta(\mathbf{r}, \theta) = \Delta_0 \exp(i\theta) \Psi(\mathbf{r}). \quad (2)$$

Here,  $\Psi$  denotes a factor which covers the spatial dependence of the pairing potential in general. Since we are only interested in the main qualitative aspects of the local DOS, namely, if the zero-energy spectral weight at

the surface is suppressed or increased, we may take the modulus of  $\Psi$  to be constant [2, 17, 18]. For the calculation of physical properties, the Riccati equations (1) have to be integrated numerically using proper starting values in the bulk. The local DOS, which is already normalized to the DOS in the normal state, is then achieved by an integration over the Fermi surface. In terms of the coherence functions  $a$  and  $b$ , we have

$$N(\mathbf{r}_0, E) = \int_0^{2\pi} \frac{d\theta}{2\pi} \text{Re} \left[ \frac{1 - ab}{1 + ab} \right]_{i\epsilon_n \rightarrow E+i\delta}, \quad (3)$$

where  $E$  denotes the quasiparticle energy with respect to the Fermi level and  $\delta$  is an effective scattering parameter that corresponds to an inverse mean free path. For all numerical calculations, we fix this value as  $\delta = 0.1\Delta_0$ .

In order to study the basic effect of chirality we consider a magnetic field applied along the  $z$ -axis at the surface, represented by a nearly homogeneous vector potential  $\mathbf{A}$ . We choose the real gauge, i.e. the spatially dependent part  $\Psi$  of the pairing potential is taken to be real. Since we additionally assumed a spatially constant modulus of the pairing potential, it is possible to get analytical solutions for the coherence functions  $a$  and  $b$  in this case, which also allows to examine the corresponding behaviour of the local DOS analytically. Directly at the surface, we get

$$N(E) = 2\text{Re} \left\langle \frac{1}{1 + a_{in}b_{out}} \right\rangle_{i\epsilon_n \rightarrow E+i\delta} - 1 \quad (4)$$

with  $a_{in} = s\Delta_0 e^{i(\pi-\theta)}$ ,  $b_{out} = s\Delta_0 e^{-i\theta}$  and the abbreviation  $s = 1/(\tilde{\epsilon}_n + \sqrt{\tilde{\epsilon}_n^2 + \Delta_0^2})$ . Furthermore,  $\langle \dots \rangle$  denotes angular averaging, which we may restrict to outgoing angles  $-\pi/2 \leq \theta \leq \pi/2$  only. This directly yields

$$N(E) = 2\text{Re} \left\langle \frac{1}{1 - (1 - 2\tilde{\epsilon}_n s)e^{-2i\theta}} \right\rangle_{i\epsilon_n \rightarrow E+i\delta} - 1. \quad (5)$$

Expanding the zero-energy DOS in orders of the vector potential  $\mathbf{A}$ , we obtain in the clean limit of  $\delta \rightarrow 0^+$

$$N(E=0) = 1 + \frac{e v_F}{c \Delta_0} A_y + \dots \quad (6)$$

Physically, this result displays the influence of a Doppler shift due to a superfluid velocity on the local quasiparticle spectrum. In terms of chiral surface states [30, 31], the Doppler shift leads to a change of the slope of the quasiparticle dispersion [ $\epsilon(k_y) = \Delta_0 k_y / k_F$ ], which has a direct impact on the corresponding DOS. As is seen from the result on the right-hand side of Eq. (6), the term of the vector potential, which belongs to the direction perpendicular to the chirality of the  $p$ -wave superconductor, survives in linear order. This is in contrast to other superconductors like  $s$ -,  $d$ - or  $p$ -wave superconductors without chirality, where we obtain similarly

$$N(E=0) = C + \langle F(\theta) \sin \theta \rangle A_y + \dots \quad (7)$$

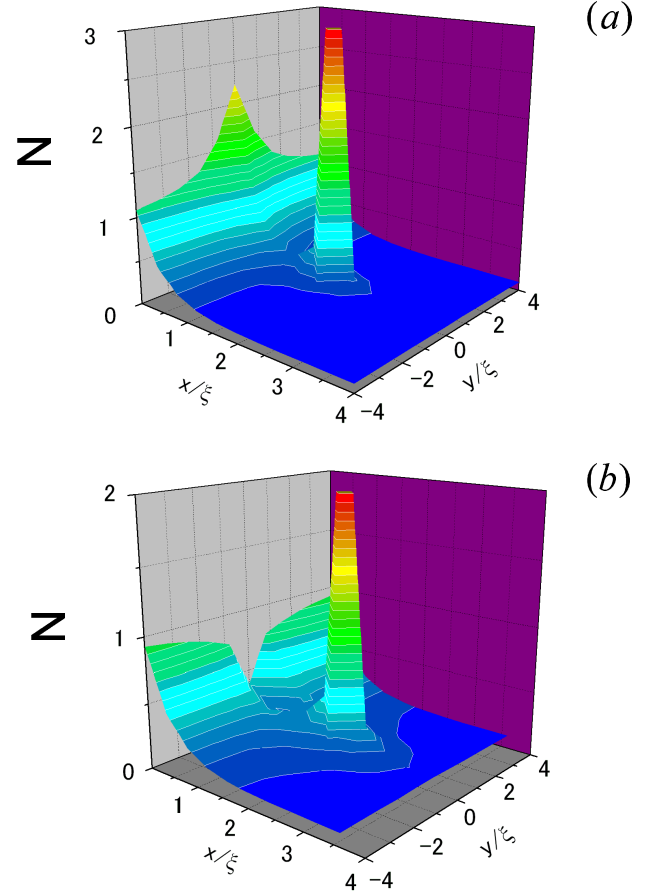


FIG. 1: (color online)  $N(E=0)$  in a chiral  $p$ -wave superconductor in the presence of a vortex, which is situated at a distance of  $x_v = 2\xi$  from the surface. The vortex and the  $p$ -wave state have (a) the same chirality and (b) the opposite chirality. Clearly, there is a remarkable difference in the quasiparticle spectral weight at the surface ( $x=0$ ) behind the vortex, showing a strong increase in (a) and a suppression in (b).

with a constant  $C$  and a function  $F$  which satisfies  $F(\theta) = F(-\theta)$ . Thus, after angular averaging, the coefficient of the linear term vanishes in these cases, reflecting the presence of inversion symmetry with respect to the  $x$ - $y$  plane. Since an applied magnetic field is related to the vector potential as  $B_z = \partial_x A_y$ , the zero-energy DOS in the chiral  $p$ -wave superconductor depends on the sign of the magnetic field. Applying a weak magnetic field along the chirality direction suppresses the zero-energy bound states, while applying it in the opposite direction leads to a zero-energy peak of the surface DOS. It is important to realize that the derived Eq. (6) qualitatively implies this chirality sensitive effect also for the case of a more general vector potential. Especially, this chirality effect is remarkable in the presence of a vortex near the surface.

As a next example of the chirality effect, we study the

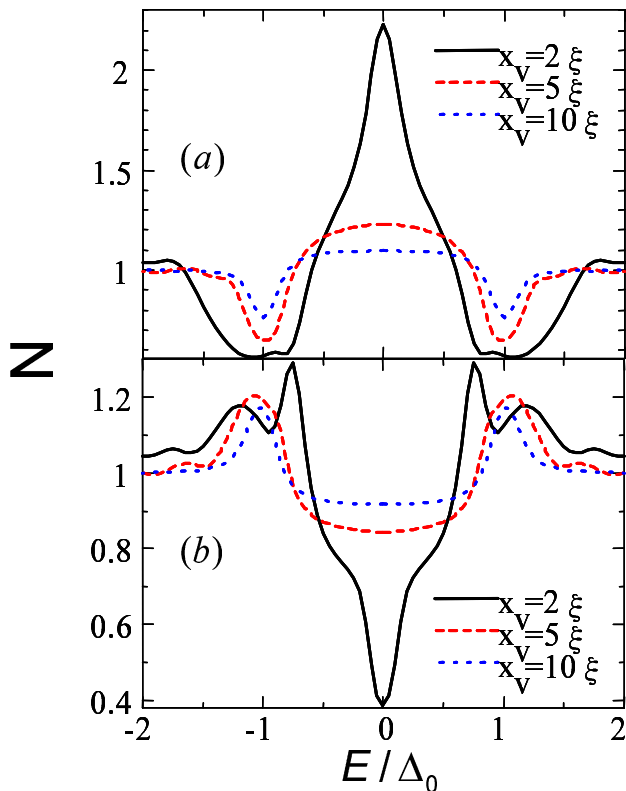


FIG. 2: (color online) Local DOS at the point  $x = y = 0$  for different vortex to boundary distances  $x_V$  as a function of energy. Again,  $p$ -wave state and Abrikosov vortex have the same chirality in (a) and the opposite in (b), resulting in peak and dip structures, respectively, around zero energy.

case of a single Abrikosov vortex line parallel to the  $z$ -axis. For such a vortex in the bulk of a chiral  $p$ -wave superconductor, the interplay between vorticity and chirality has been intensively studied already [32]. In the present work, however, we focus on important surface effects, which appear at the boundary due to the presence of the vortex. We assume the vortex to be at a distance  $x_V$  from the boundary. With  $y$  denoting the coordinate along the boundary, the vortex shall sit at  $y_V = 0$ . For convenience, we set the vector potential  $\mathbf{A} = 0$  and incorporate the vortex by a properly chosen phase factor  $\Psi(\mathbf{r}) = e^{i\Phi(\mathbf{r})}$  instead, which (in standard complex notation  $z = x + iy$ ) is given by [17, 18]

$$e^{i\Phi(\mathbf{r})} = \frac{z - z_V}{|z - z_V|} \cdot \left( \frac{z - \bar{z}_V}{|z - \bar{z}_V|} \right)^* \quad (8)$$

Here, the first factor is the phase of a single vortex at position  $z_V$ , while the second factor corresponds to a virtual antivortex placed at the mirrored position  $\bar{z}_V = -x_V$ , which ensures the correct implementation of boundary conditions. In the following, we consider two different cases: The chiralities of the  $p$ -wave state and the vortex are the same (a), or opposite (b). The latter case is implemented by a complex conjugation of the phase factor

given in Eq. (8), replacing the vortex by an antivortex and vice versa. Our results for the corresponding local zero-energy DOS near the surface of the  $p$ -wave superconductor are shown in Fig. 1. The Abrikosov vortex position is fixed at a distance of  $x_V = 2\xi$  from the boundary with  $\xi = \hbar v_F / \Delta_0$  denoting the coherence length. Apart from zero-energy bound states in the vortex core, there are also bound states at the surface of the  $p$ -wave superconductor. Far away from the vortex, these surface bound states have the spectral weight of  $N(E = 0) = 1$  [cf. Eq. (6)]. However, as can be seen quite clearly in Fig. 1, the local DOS drastically changes in a shadow-like region behind the vortex. If vortex and  $p$ -wave state have the same chirality, the bound states are strongly enhanced (a), for opposite chirality they are suppressed (b). The latter suppression resembles a similar effect appearing in  $d$ -wave superconductors [17, 18].

In Fig. 2, we show the local DOS at the point  $x = y = 0$  for different vortex to boundary distances  $x_V$  as a function of energy. Around zero energy, we find a sharp peak or dip structure, respectively, again depending on the chirality. These structures get less pronounced, when the vortex distance is increased, nevertheless they persist. Moreover, the quasiparticle spectrum starts to exhibit some kind of mirror symmetry around the value  $N(E) = 1$  for the two different chiralities. Note that this is in qualitative agreement with Eq. (6) since after a transformation to the real gauge we eventually have  $A_y > 0$  due to the vortex, leading to the strong increase of Andreev bound states at the surface for the same chirality. The result for opposite chirality is obtained due to  $A_y < 0$  for the antivortex, accordingly. It is worth noting that the modification of the surface quasiparticle states due to the presence of vortices has an effect on the force acting on vortices near the surface. An increase of the DOS leads to a repulsion of the vortex from the boundary towards the bulk, whereas a decrease results in an attraction towards the boundary. Thus, in both cases the Bean-Livingston barrier would be modified, which influences the escape and entrance of vortices to the superconductor [33, 34].

Our results allow us to propose a chirality sensitive test on superconductors, based on well-established experimental techniques, which are capable of indicating the weight of Andreev bound states, for example tunneling conductance experiments or low-temperature scanning tunneling spectroscopy [35]. For a chiral superconductor, it is expected to observe a suppression of the zero-energy DOS at the surface, when a weak magnetic field is applied parallel to the chirality. Inverting the field, however, leads to an enhancement of the DOS. In this way chirality could be detected. This unusual reversal effect does not appear in non-chiral superconductors. Moreover, the experiment allows us to detect the chirality and possibly even the domains of different chirality, if domain walls reach the surface.

In summary, we have studied the DOS at the surface of a chiral  $p$ -wave superconductor in the presence of a magnetic vector potential due to, for example, an applied magnetic field or an Abrikosov vortex. We clarified that the weight of low-energy surface bound states gets either suppressed or increased, depending on the orientation of both chirality and magnetic field. Due to this chirality-sensitive effect on the Andreev bound states, a setup to test the chirality of an unconventional superconductor could be accessible experimentally. Furthermore, for vortices this effect also has a chirality selective influence on the Bean-Livingston barrier which could give rise to a different escape rate of vortices from the two kinds of chiral domains.

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